Leaders Booklet

Subitizing: laying the foundations for number sense

An Action Research Project

devised by

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Natural Maths

The theme for this project is:

> Does an emphasis on subitizing promote improved number sense and computation skill?

Subitize: means 'to suddenly know'. When we look at a small collection we suddenly know how many there are without needing to count.

Much has been written about our innate ability to subitize, compare and quantify before we can even count and yet we gloss over this natural ability when laying down foundations for number sense. We know much about the brain and how it has evolved. It is true to implement a more brain based approach to number. *"The human brain is a five-star pattern recogniser." "The brain's ability to detect patterns and make associations is one of its greatest strengths."*[1]

Clements[2] says that subitizing may be the developmental prerequisite skill necessary to learn counting.

Typical action research projects follows steps similar to those shown below.

- Select a question or concern to look at in depth in the classroom (see proposed questions)
- Review the research findings available (some summaries provided along with references)
- Collect pre data (activities provided)
- Create a teaching sequence and observation schedules (ideas and proformas provided in this pack)
- Collect and interpret data from observations and student interviews (ideas provided)
- Use data collected to inform teaching on the job
- Post action data collection
- Summarise and share experiences and data collected

This is the proposed model for this project which will be classroom-based and

Note: sharing at each stage is crucial to this process

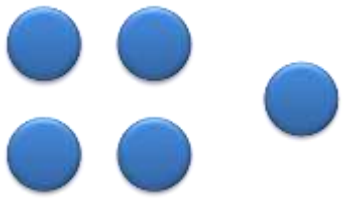
[1] Sousa, D. A. (2008) How the brain learns mathematics, Corwin Press

[2] Clements, D. H. (1999) Subitizing. What is it? Why teach it? Teaching Children Mathematics, March 1999 Edition

Prentice and Starkey[3] have shown that babies as young as 16 – 30 weeks can subitize. Babies can notice the differences between a screen with 2 dots and a screen with 3 dots. During the first few months of life babies notice constancy of objects and can detect changes and differences in numerical quantities. At twenty months a toddler can look at two collections and point to the one that has most, for instance, when there are 3 blueberries in one hand and 5 in the other, assuming that they like blueberries will select the hand holding most blueberries.

Animals also subitize and in the wild their life may depend on subitizing to estimate and compare group/pack size. Whether to attack or flee may be a matter of life and death and the decision is dependent on detecting and comparing quantity. It is important to note that animals cannot count but do have a basic number sense or numerosity. Numerosity is the term for quantifying, comparing and subitizing before actually being able to count. Numerosity may be the prerequisite for learning to count and to work effectively with number. It is the basis for later number sense.

Doug Clements (see previous page) differentiates between perceptual subitizing, just knowing and conceptual subitizing which requires the brain to carry out some processing to determine the quantity in a collection. In the example below for instance the brain will possibly notice the 4 dots and the 1 to the side and automatically combine the two groups so that 5 is identified almost unconsciously.



There are strong suggestions that all later maths is built on the ability to subitize.

When looking at a collection as in the example above the brain can see the whole collection and then identifies the parts and combines them to quantify the whole. This natural decomposition of numbers naturally is the first developmental milestone in the concept of *altogether*. It also lays down visual patterns that make mental computation strategies visible and accessible. Doubles and near doubles are good examples of patterns that can be spotted through subitizing and about which number talk can develop.

From subitizing unitizing can develop. Unitizing refers to making equal sized groupings to speed up the counting process. When asked to draw 23 honey ants such that there is no need to count each ant to find out how many are drawn can be made easier by drawing ants in subitizable groups of 5 to speed up counting. The multiplicative nature of 20 will be seen in the drawing of 4 groups of 5. This early experience is laying the foundation for understanding the multiplicative property and learning multiplication facts with understanding. Based on the body of research about subitizing it seems important to begin with the innate pattern spotting ability to lay a firm foundation for later mathematics. We can't build a house on a wobbly foundation and neither can we build a strong number sense without a firm foundation.

[3] In the Sousa book referenced on the previous page.

Subitizing is considered to be:

- innate, we are genetically set up to recognise patterns in quantity
- the basis of numerosity, quantifying and comparing the size of a group of objects
- the foundation for estimation and comparison of numbers
- the foundation of most later mathematics
- a predictor of how well a student will progress mathematically, indeed, "subitizing may be the developmental prerequisite skill to learn counting."[4]

Subitizing leads to:

- being able to identify, talk about and work with numbers in a general way without having to carry out precise calculations
- developing ideas of magnitude and making informed estimates, e.g., 'there are more than 5 but less than 10' or 'there are more than 20 but less than 40'
- being able to look at and notice without needing to count, e.g., looking at 3 muffins and knowing if they all have the same number of M&Ms
- conceptually (and hence speedily) combining two groups of objects to say how many altogether

There is a short video segment, APY subitizing introduction, in which Ann introduces the way in which she would begin working with students on the subitizing theme.

[4] Warren, E., Cole, A. and de Vries, E. (2009) 'Closing the gap: Myths and truths behind subitisation' Australasian Journal of Early Childhood 34 (4) Dec, pp46-53

In this section we look at refinements that could be made to the original statement of intent:

> Does an emphasis on subitizing promote improved number sense and computation skill?

This general statement can be broken down into parts such as:

> Can subitizing be strengthened through practice?
>
> Can we make better use of the brain's innate patterning ability to improve number sense?
>
> If you model risk taking, estimation and enjoyment as you work with the students together on subitizing, does this flow through to their experience of subitizing?
>
> Do the student respond better to subitizing with dots (as on the dot cards) with cultural images (such as collections of ants) or topical images (such as soccer balls)?
>
> Do the students naturally begin to identify irregular patterns and integrate efficient strategies such as count on, double etc.?
>
> Does subitizing promote improved number sense, as in the ability to:
>
> - quickly identify how many in a small group
> - make visual comparisons of group size
> - recreate displayed patterns
> - use patterns for early computation (conceptual subitizing)

Further refinements could include:

> Will reinforcing subitizing help develop reliable and efficient counting strategies?
>
> Can subitizing be strengthened through problem solving?
>
> Does the modelling of enjoyment of maths make it contagious?
>
> How does awareness of the brain's innate sense of patterning open up potential for acquiring number sense?
>
> Does teaching intuitive rather than counter-intuitive number (e.g. rote learning) strategies enhance learning?
>
> Does using context for subitizing make a difference (dots or honey ants)?

The final statement for your action research project should be established through discussion with members of the group of teachers that you are working with.

As you work through this subitizing action research project it is expected that

- Five students (across the range of the class) will be selected to monitoring growth throughout the unit and that these same students will be involved in the pre and post-test.
- Time will be set aside for subitizing every day and for making observational notes.
- Observation notes will be made to help monitor change over time and to plan next moves.
- AEWs will be involved with playing the games with the students as well as in creating subitizing resources of cultural or topical interest, this can be part of sitting with students as they also create their own subitizing resources
- Vocabulary posters supported with images will be visible on the walls in classrooms and students will be immersed in the correct vocabulary associated with the unit.
- Collection of anecdotal observations and data from student work samples will be collected and used for sharing and or moderation within school and with others via Polycom.
- Games and resources will be sent home for take home activities once students are fully confident with them.

There is also a video sequence, **Teacher Interview**, which shows Ann talking with a teacher about the diagnostic information that emerged from observing students as they played with the subitizing cards.

This video shows how observations can lead to informed decisions as to what would be appropriate action to take next with the students – an important expectation for this action-research project.

The first set of dot cards are for the perceptual subitizing games and activities. The dot patterns used

It is important to be intentional and systematic to create a developmental sequence matched to student's current understandings. The Top 5 allows for a short developmental sequence to be planned and expressed in child friendly language.

My first Subitizing TOP 5

Start Date | Finish Date

- I can quickly match subitizing cards.
- I can quickly say how many dots on a card without counting.
- I can compare 2 cards and say one has more than, less than or the same as the other card.
- I can play subitizing games.
- I can use subitizing to solve a problem.

Congratulations

A job well done!

Quick Check

1. Can you find me the card that matches this card?.
2. How many spots on this card? Did you have to count the spots?
3. Does this card have more spots than that one? Which of these two cards has the most spots?
4. Let's play a game of subitizing. Which card does the dice match now?
5. I've chosen a card. Can you think of questions you could ask me to find out how many spots my card has?

The Top 5 needs to be achievable in a 2 - 3 week time frame. Clearly identified learning intents bring many benefits for teachers, students and others alike. As a teacher having a concise targeted set of intents provides indicators for observations and identifies the next nudge. Formative assessment is easier to carry out when you know what you are looking for and know what questions and nudges to make next.

Because the Top 5 is shared with the students and others it also means that everybody is on the same page and can speak the same language and push the same ideas. Students know what it is they are expected to learn and feedback can be given on the job to let students know how they are going. It is also possible to articulate for the students what they have been successful at, for instance,

> "Last week you couldn't subitize 4, 5 or 6 but today you were really fast at it. Are you pleased with yourself, you should be?

Even better is when a student identifies for themselves when they have mastered one of the Top 5s and shares their success with others.

The Quick Check allows you or a teacher's aide or AEW to find out exactly what the students can do and do know prior to and or after the unit of work.

The resources provided enable the teacher to run a variety of activities. These are described below.

The purpose of mental routines is to develop the language that you want the students to acquire. In this case, the language is mainly comprised of number names and comparative terms such as *more than* or *between*.

The students gather in an open space and all are given their own laminated card or hands-on materials and suitable writing materials. We call these mini-whiteboards, as normal felt pens can be used to ring or mark ideas on the laminated cards.

The teacher begins by posing simple, **closed** questions that enable everyone to be successful. Soon, the questions change to **open** questions, where more than one answer can be found. This enables students to begin to work at their own level. Finally, in the **flip** questions, it is the students who ask the questions, trying to determine a solution to the problem that the teacher has posed

The mental routines make an excellent lesson starter as they arouse enthusiasm and encourage the students to feel part of a learning community. They need last no more than ten minutes, but in that time every student has been engaged and has been challenged to take risks with their current understanding.

Where it becomes clear that students need to acquire or consolidate a particular strategy, the strategy lesson plans suggest ways in which the students can engage in meaningful learning. They provide the opportunity for students to share their thinking and, as a community of learners, choose the strategy for achieving a given outcome that suits their current level of development.

Target strategies, needs based, are identified for these lessons so that the focus is clear to all and so that prompts and suggestions are matched to the identified needs. Often a stimulus is provided for discussion and to allow students to articulate their thinking about a particular aspect. An emphasis on vocabulary and the correct use of mathematical language to explain thinking needs to be a clear focus for the strategy lessons. Ensure that during the lesson you are immersing your students in the vocabulary that will allow them to deeply understand the concept being explored.

Students do need practice but it needs to be purposeful and engaging practice which is why most strategy lessons result not in pages of exercises but in:

- ✓ Student designed and made posters, pages for a class book or puzzles that they can try on each other
- ✓ Games that enable them to have fun while they develop fluency in the target strategies.

The reflection at the end of a strategy lesson is very important and offers an opportunity for the teacher to formalise the learning that has taken place.

NOTE: Skill getting is only part of the process; the next step has to be to see what learnings the students bring to the next problematized situation. Skill using is the goal.

We use the term “problematized situation” to describe the type of activities that will allow students to engage with realistic (to them) situations as described in the research from the Freudenthal Institute. The situations provide the kinds of challenges that encourage students to construct their own ideas, strategies and mathematical understandings as they grapple with them. The students, as described earlier, are developing their own mathematical tools, which can be formalised by the teachers when appropriate.

Problematized situations have multiple entry points and many methods of solution. If the numbers are too hard, they can be reduced; if they are too easy, they can be increased. Some students will draw pictures or act out the solution with objects whereas others may use a more symbolic approach using numbers or tallies. Some will present solutions in an organised fashion whereas others will be more muddled. It is the sharing and reflecting on the range of strategies that will broaden the possibilities for the students and allow them to enter into mathematical thinking from their very first experiences. The focus in the primary classroom is shifting towards an emphasis on mathematical reasoning and problem solving in a true sense. This new focus helps students learn to describe, compare and discuss their multiple approaches to solving real problems. In the classrooms where we have been working, we have noted students have an engagement with the problems and an increased interest in maths along with a really firm conceptual understanding.

In the busy classroom the end of lesson approaches all too quickly and as a result the reflection is often neglected, and yet the reflection is the most important part of the lesson. It is the time when the students use mathematical language to explain what they have done. It is the time when students see that there are many strategies for solving problems and that some are more effective than others. It is also the time when the teacher can formalise a particular idea, concept or process and scaffold the students to the next level. In fact there are some who go so far as to say that if you don't do a reflection then the students will probably retain nothing. The development of a community of learners who share, listen and learn from each other is at the heart of this approach to mathematics. The reflection time sets up the mathematical culture of the classroom with its tight-knit community of learners. It allows for mathematical mind journeys and adds to the excitement of learning mathematics.

The principles of rigorous reflection are:

- the identification of a range of strategies to share and discuss
- the use of one or more errors to show the value of checking results and of developing a fix-up strategy
- celebrating risk-taking, inventiveness, mathematical reasoning and learning from mistakes
- building on, extending and presenting more formal methods of recording as students demonstrate readiness for them
- positive, constructive feedback with a focus on feed forward – what you will do next time.

Through the dialogues and participation of all students in the class, the reflection stage becomes crucial to the development of a community of learners, through which active involvement in learning mathematics is successfully fostered.

There are many opportunities during the day to model purposeful counting and counting on or back. Such opportunities allow students to hear and be immersed in the counting patterns that we want them to be able to use. Examples include.

- ✓ counting the steps to the library – "Take big steps as you go and small steps as you come back, what changes?"
- ✓ count students lined up in 2s or sitting in 5s
- ✓ count how many students have arrived and count on as more arrive or as some leave
- ✓ timing, how far we can count in 1s, 2s, 5s, 10s while we wait for something to happen, e.g., student to collect the pencils
- ✓ count down from 100 in 1s, 2s, 5s, and 10s as you wait for the bell to go.

Our intention in this book is to provide the background research that can inform how we should teach place value as well as mental routines and problematized situations that will support your teaching of this crucial topic.